

Characterization of the temperature oscillation technique to measure the thermal conductivity of fluids

P. Bhattacharya^a, S. Nara^a, P. Vijayan^a, T. Tang^a, W. Lai^a, P.E. Phelan^{a,*},
R.S. Prasher^b, D.W. Song^b, J. Wang^b

^a *Arizona State University, Department of Mechanical and Aerospace Engineering, Building ECG, RM 346,
Tempe, AZ 85287-6106, United States*

^b *Assembly Technology Development, Intel Corporation, 5000 W. Chandler Boulevard, Chandler, AZ 85226, United States*

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Abstract

The temperature oscillation technique to measure the thermal diffusivity of a fluid consists of filling a cylindrical volume with the fluid, applying an oscillating temperature boundary condition at the two ends of the cylinder, measuring the amplitude and phase of the temperature oscillation at any point inside the cylinder, and finally calculating the fluid thermal diffusivity from the amplitude and phase values of the temperature oscillations at the ends and at the point inside the cylinder. Although this experimental technique was introduced by Santucci and co-workers nearly two decades ago, its application is still limited, perhaps because of the perceived difficulties in obtaining accurate results. Here, we attempt to clarify this approach by first estimating the maximum size of the liquid's cylindrical volume, performing a systematic series of experiments to find the allowable amplitude and frequency of the imposed temperature oscillations, and then validating our experimental setup and the characterization method by measuring the thermal conductivity of pure water at different temperatures and comparing our results with previously published work.

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1. Introduction

The two main techniques to measure the thermal conductivity of fluids are the transient hot wire technique [1] and the temperature oscillation technique [2]. The transient hot wire technique was introduced in 1974 [1]. In this technique a cylindrical fluid volume is heated electrically using a current-carrying metallic wire stretched along the axis of the fluid volume. The differential temperature rise of the wire is calculated based on the change in the electrical resistance of the wire at different time points, and then the temperature differentials are plotted against the natural logarithm of the time. This plot is expected to have a linear region, from the slope of which the thermal diffusivity of

the fluid can be calculated. The theory behind this technique is based on an infinitely long linear heating element. However, a wire of a finite length and a finite diameter is used to act as the infinitely long linear heating element. That and some other factors such as the finite thermal diffusivity of the wire, the temperature jump in the fluid adjacent to the wire, the existence of an outer boundary of the cell, and natural convection, introduce inaccuracies in the measurement of the thermal diffusivity of the fluid. J. Kestin and his colleagues found methods to account for these factors [3,4]. However, these corrections are valid only after one has used a sufficiently thin and long wire of sufficiently high thermal diffusivity, and the duration of the measurement is neither very short nor extremely long. Thus, the already complicated experimental setup, with so many hard-to-satisfy constraints, make the transient hot wire technique somewhat less attractive. It should be

* Corresponding author. Tel.: +1 480 965 1625; fax: +1 480 965 1384.
E-mail address: phelan@asu.edu (P.E. Phelan).

Nomenclature

B^*	ratio of complex amplitudes	τ	non-dimensional time
G	period of oscillation (rad)	ω	angular velocity of oscillation (rad s ⁻¹)
T	temperature (°C)	ξ	non-dimensional distance
T_m	mean temperature of oscillation (°C)		
t	time (s)		
t_p	time period of oscillation (s)	<i>Subscripts</i>	
u	amplitude of oscillation (°C)	0	value at $x = 0$
x	distance (m)	L	value at $x = L$
<i>Greek symbols</i>			
α	thermal diffusivity (m ² s ⁻¹)		
ν	kinematic viscosity (m ² s ⁻¹)		

noted here that we do not intend to present a detailed analysis of the relative advantages and disadvantages of the transient hot wire technique and the temperature oscillation technique in this work. Instead, this paragraph is intended to give a brief description of why we did not choose the transient hot wire technique.

On the other hand, the simplicity of the temperature oscillation technique makes it more appealing. In this technique, a temperature oscillation is imparted from the two ends of a cylindrical fluid volume. As the oscillation travels along the length of the fluid cylinder, its amplitude decreases and phase lags and at the center, the amplitude reaches the minima and the phase lags the most. The amplitudes and the phases of the temperature oscillations are measured at different points inside the fluid and the thermal diffusivity of the fluid is calculated from those measured values. The theory behind the temperature oscillation technique has been available in the literature since 1984 [2]. According to Santucci et al. [5] this method was first proposed by none other than A.J. Angstrom in 1863. However, the technique was first used to measure the thermal diffusivity of fluids only in 1986 [5]. After that very few studies [6–8] have been reported where this technique has been used. Both [6] and [8] explain the experimental setup to some extent. There is no literature available, though, which gives a detailed analysis and guidance about what configuration, shape and size of the experimental setup to choose, and also what values of amplitude and frequency of the temperature oscillation should be utilized. This is probably the main reason why people have preferred other techniques to the temperature oscillation technique to date. In order to evaluate the feasibility of applying this approach to measure the thermal conductivity of nanofluids [9] and other novel engineered fluids, we critically analyze its limitations, and through a careful series of controlled experiments we enable the applicable range of imposed temperature oscillation frequencies and amplitudes to be ascertained.

Another popular method for measuring the thermal conductivity of materials is the 3- ω method, first proposed

in 1987 [10] and further developed [11] in 1990 mainly to measure the thermal conductivity of dielectric solids. This technique is similar to the transient hot wire technique. The sample is heated with a thin and long hot wire or a hot strip. An alternating current is passed through the wire or the strip to heat it up. The main difference between the transient hot wire and the 3- ω method is that in the first the time response of the temperature differential is used to calculate the thermal conductivity, whereas in the latter the frequency response of the temperature differential is used to do the same [11]. The best part of this technique is that it is insensitive to the black-body radiation. One requirement of this technique is that the sample volume should be small. This in turn means that very small length-scale measurements can be made using this technique, and indeed it is widely used to measure the thermal conductivity of solids at very small length scales [12–14]. Although this technique was used to measure the thermal conductivity of liquids as early as 1991 [15], it did not become very popular for the measurement of the liquid thermal conductivity due to some unknown reason. The first among the problems with this technique is the requirement for a “long” and “thin” heating wire similar to that required for the transient hot wire technique and the uncertainties associated with that. The second is the requirement of a small sample volume, which for our purposes of measuring the thermal conductivity of nanofluids, would not be very suitable because the “wall effect” might be large on the particles and hence can affect the property of the nanofluids.

2. Theory behind the temperature oscillation technique

The measurement of the thermal diffusivity and the thermal conductivity is based on the solution of the transient heat conduction equation [6–8]:

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T \quad (1)$$

where α is the fluid thermal diffusivity. The cylindrical fluid volume considered for analysis is shown in Fig. 1. At the

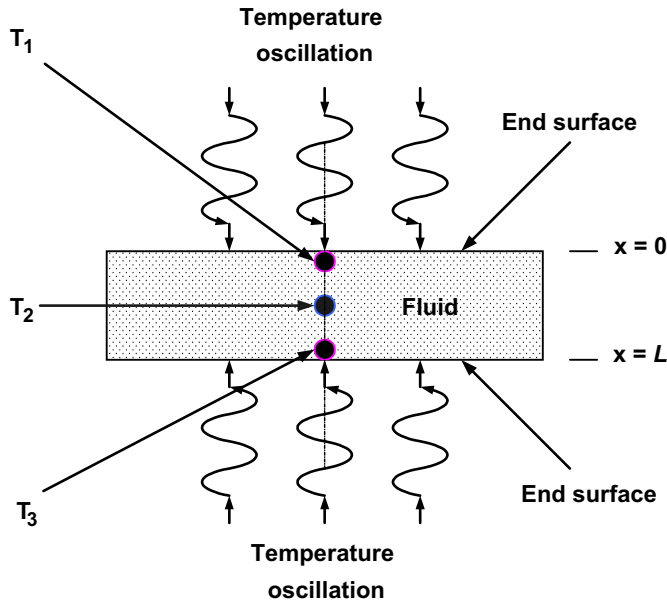


Fig. 1. Conceptual diagram of the cylindrical fluid volume at the two ends of which the temperature oscillation is applied, and T_1 , T_2 , and T_3 are the three thermocouples.

two ends of the cylinder, periodic temperature oscillations are generated with an angular frequency, ω , given by

$$\omega = \frac{2\pi}{t_p} \quad (2)$$

where t_p is the time period of the oscillation. For simplicity, here we assume and in Section 4 we justify that there is no heat transfer in the radial direction, which means the heat transfer is only in the vertical direction, and hence we choose to use the one-dimensional form of Eq. (1). Now we define the non-dimensional space, ξ , and time, τ , coordinates as

$$\xi = x \cdot \sqrt{\frac{\omega}{\alpha}} \quad (3)$$

$$\tau = \omega \cdot t \quad (4)$$

Using Eqs. (3) and (4), we can non-dimensionalize Eq. (1) as,

$$\frac{\partial^2 T}{\partial \xi^2} = \frac{\partial T}{\partial \tau} \quad (5)$$

For the general case of imposed oscillations of the same frequency but with different amplitude and phase at $x = 0$ and at $x = L$, the boundary conditions are given by

$$T(\xi = 0, \tau) = T_m + u_0 \cos(\tau + G_0) \quad (6)$$

$$T\left(\xi = L\sqrt{\frac{\omega}{\alpha}}, \tau\right) = T_m + u_L \cos(\tau + G_L) \quad (7)$$

where T_m is the mean of the imposed temperature oscillations, u_0 the amplitude of oscillation at $x = 0$, u_L the amplitude of oscillation at $x = L$, G_0 the phase of the oscillation

at $x = 0$, and G_L the phase of the oscillation at $x = L$. Under steady periodic conditions, the solution of Eq. (5) with boundary conditions given by Eqs. (6) and (7) can be obtained by using the method of Laplace transforms [7]. The solution can be written in complex form as [7]:

$$T(\xi, \tau) = T_m + \frac{u_L e^{iG_L} \sinh(\xi\sqrt{i}) - u_0 e^{iG_0} \sinh[(\xi - \xi_L)\sqrt{i}]}{\sinh(\xi_L\sqrt{i})} \cdot e^{i\tau} \quad (8)$$

The ratio of the complex amplitude at $x = 0$ to that at any point along the length, B_0^* , is given by:

$$B_0^* = \frac{u_0 e^{iG_0} \sinh(\xi_L\sqrt{i})}{u_L e^{iG_L} \sinh(\xi\sqrt{i}) - u_0 e^{iG_0} \sinh[(\xi - \xi_L)\sqrt{i}]} \quad (9)$$

The ratio of the complex amplitude at $x = L$ to that at any point along the length, B_L^* , is given by:

$$B_L^* = \frac{u_L e^{iG_L} \sinh(\xi\sqrt{i})}{u_L e^{iG_L} \sinh(\xi\sqrt{i}) - u_0 e^{iG_0} \sinh[(\xi - \xi_L)\sqrt{i}]} \quad (10)$$

The real measurable phase shift, ΔG , and the amplitude ratio, r_u , can be expressed as:

$$\Delta G = (G_j - G(x)) = \arctan \left[\frac{\text{Im}(B_j^*)}{\text{Re}(B_j^*)} \right] \quad (11)$$

$$r_u = \frac{u_j}{u(x)} = \sqrt{[\text{Re}(B_j^*)]^2 + [\text{Im}(B_j^*)]^2} \quad (12)$$

where $j = 0$ or L . By measuring ΔG and r_u , α of the fluid can be obtained by solving either Eq. (11) or (12). Once we know α , by knowing the density and the heat capacity of the fluid, we can calculate the fluid thermal conductivity, k .

3. Experimental setup

The experimental setup is shown in Fig. 2. Fluid is placed inside the cylindrical test chamber, the two ends of which are closed with the help of the reference plates which act as heat spreaders. We use two Peltier devices, i.e., thermoelectric heaters/coolers, at the two ends of the test chamber on the outer surface of the reference material as the heating/cooling devices. These are used to supply the input temperature oscillations. The two Peltier devices are electrically connected in series so that they carry the same current and consequently have the same temperature at the load side (the reference material side). The entire system is held together by the two endplates. The endplates also act as heat spreaders on the heatsink sides of the Peltier devices. Two copper heat exchangers are used as heatsinks, and water is used as the heat transfer fluid in the heatsink system. There are five thermocouples connected to five different points in the system as shown in Fig. 2. One of them

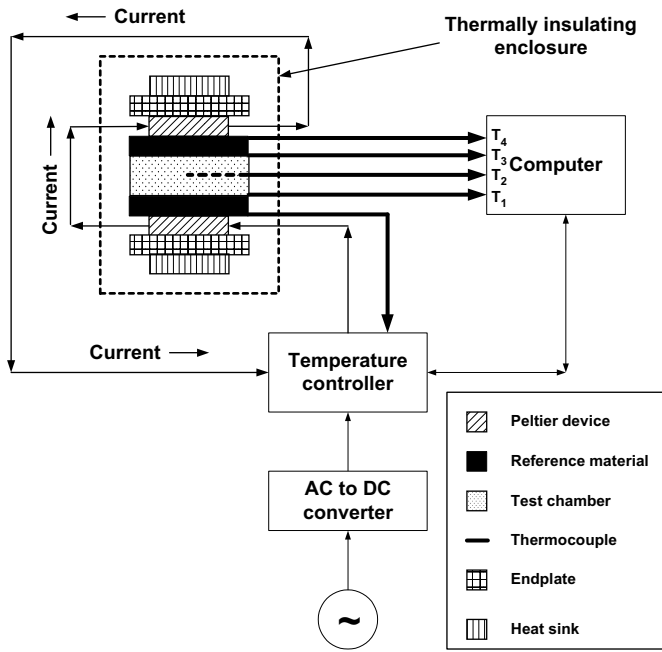


Fig. 2. Schematic diagram of the experimental setup.

reads the temperature of the load side of one of the Peltier devices, and sends the data to the temperature controller. The temperature controller, in turn, communicates that information to a Labview program and based on that, receives commands about how much current needs to be supplied to the Peltier devices, and in which direction the current should flow. Thus, any desired temperature is achieved at that particular Peltier device surface. Because of the series connection, the same temperature is also attained at the load surface of the other Peltier device. The other four thermocouples are directly connected to the data acquisition device. One of them measures the temperature at the center of the fluid volume (T_2), two measure the temperatures at the center of the end surfaces of the fluid volume (T_1 at $x = 0$ and T_3 at $x = L$), and the fourth one measures the temperature at the load side of the second Peltier device (T_4).

4. Optimum size of the test chamber

The theory requires that the heat conduction be one-dimensional along the length of the cylindrical test chamber. It can be achieved in two different ways. First, the diameter-to-length ratio of the chamber should be greater than 1. Second, the reference plates, which act as heat spreaders at the two ends of the cylinder, should have high thermal conductivity, and they should be sufficiently thick to enable adequate heat distribution from the Peltier devices. In our case, the length and the diameter of our cylindrical test chamber are 5 mm and 40 mm, respectively, and our reference plates are made of 6-mm-thick aluminum plate.

The theory also requires that the only mechanism for heat transfer within the fluid to be conduction, which means in practice that natural convection must be avoided. The onset of natural convection depends on the type of fluid, the dimensions of the test chamber, and the amplitude and the frequency of the temperature oscillation. The rest of this section will address this issue.

The critical Rayleigh number, $Ra_{x,cr}$, where x is the characteristic length, decides the onset of natural convection. For vertical heat transfer between two plates its value is 1700, while for horizontal heat transfer between two plates its value is 1000 [16]. Natural convection becomes significant when Ra_x is higher than these critical values. That is one reason why we chose to use a vertical cylinder rather than a horizontal cylinder. Therefore, for all our subsequent calculations we will use $Ra_{cr} = 1700$. For the system in consideration, where the same temperature oscillation is applied at the two ends of a vertical cylindrical fluid volume, the characteristic length would be half of its length. The Rayleigh number for this system can thus be expressed as,

$$Ra_{L/2} = \frac{g \cdot \beta \cdot \Delta T \cdot \left(\frac{L}{2}\right)^3}{\alpha \nu} \quad (13)$$

where g is the acceleration due to gravity, β the volumetric thermal expansion coefficient of the fluid, ΔT the temperature difference driving the natural convection, and ν the kinematic viscosity of the fluid. If we start with a fluid of known properties, as for example water, we know g , β , α , and ν in Eq. (13). The value of the quantity $[\beta/(\alpha\nu)]$ increases with increasing temperature for almost all fluids. For the present case we use the values for water at 85 °C to account for the worst-case situation which we expect to face: $[\beta/(\alpha\nu)] = 1.21 \times 10^{10} \text{ m}^{-4} \text{ s}^2 \text{ K}^{-1}$ [16]. Thus we obtain:

$$\left[\Delta T \cdot \left(\frac{L}{2}\right)^3 \right]_{\max} = 1.405 \times 10^{-7} \text{ m}^3 \text{ K} \quad (14)$$

using $Ra_{L/2} = 1700$ so that natural convection is avoided. To estimate the individual values of ΔT and L that would satisfy the conduction equations we use the following method.

Combining Eqs. (10) and (12) for conditions $u_0 = u_L$, $G_0 = G_L$, and $x = L/2$, we can write:

$$r_u = \frac{u_L}{u_{L/2}} = \sqrt{\left[\text{Re} \left(\cosh \left[\frac{L}{2} \left(\frac{2\pi}{\alpha t_p} i \right) \right] \right) \right]^2 + \left[\text{Im} \left(\cosh \left[\frac{L}{2} \left(\frac{2\pi}{\alpha t_p} i \right) \right] \right) \right]^2} \quad (15)$$

In the present situation ΔT can be estimated as the following:

$$\Delta T = u_L - u_{L/2} \quad (16)$$

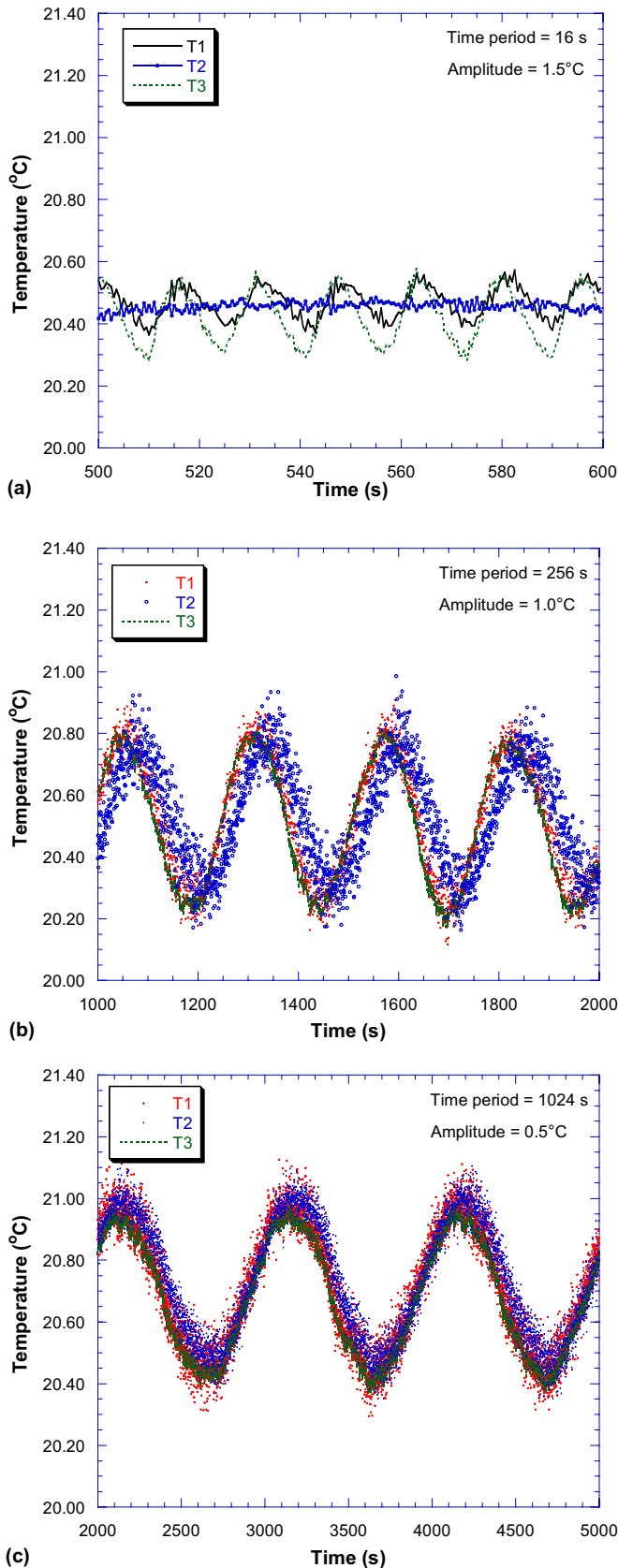


Fig. 3. (a) A no-oscillation situation at thermocouple T_2 , (b) a nice waveform with clear phase difference, and (c) a situation where there is no discernible phase difference.

Combining Eqs. (15) and (16) we obtain:

$$\frac{u_L}{u_L - \Delta T} = \sqrt{\left[\operatorname{Re} \left(\cosh \left[\frac{L}{2} \left(\frac{2\pi}{\alpha t_p} i \right) \right] \right) \right]^2 + \left[\operatorname{Im} \left(\cosh \left[\frac{L}{2} \left(\frac{2\pi}{\alpha t_p} i \right) \right] \right) \right]^2} \quad (17)$$

u_L depends on the amplitude of the input temperature oscillation. For an input amplitude of 1.0°C at the Peltier device, u_L turns out to be around 0.5°C . For the sake of the present calculation, we consider u_L to be 0.5°C . For an arbitrary value of $t_p = 256\text{ s}$ we solve Eqs. (14) and (17) simultaneously for L and ΔT using an iterative method to find $L = 8.7\text{ mm}$ and $\Delta T = 0.173^\circ\text{C}$.

This means, while conducting experiments with water at 85°C with $t_p = 256\text{ s}$ and input oscillation $= 1^\circ\text{C}$, the maximum chamber length that can be used is 8.7 mm . The effect of natural convection increases with decreasing t_p , with increasing input amplitude of the temperature oscillation, and with increasing $[\beta/(\alpha\nu)]$. This was also verified experimentally. Since both our experimental setup and the heating condition are symmetric in the vertical direction, in the absence of natural convection, temperature responses at the two ends of the cylindrical volume should be the same. If natural convection is present, due to anti-symmetric nature of the convection with respect to the central plane, a difference in the above-mentioned temperature responses would be observed. Fig. 3(b) shows the temperature response of an experiment for which the amplitude and the time period used did not allow any natural convection and consequently the temperature response at the two ends of the cylinder are identical. In contrast, Fig. 3(a) shows the time response of an experiment with a smaller time period and a greater amplitude. Natural convection took place in this case and consequently the temperature responses at the two ends of the cylindrical volume are different.

5. Data analysis and error calculation

We acquire two data points each second. Each of these data points is the average of the 500 data points we capture in half a second. We find the mean value of each of the three temperature waves, T_1 , T_2 , and T_3 , at each time point. Then we calculate the combined standard deviation of the three means over four consecutive cycles. We consider the four cycles to be in steady-state periodic condition, if the standard deviation is less than 0.07°C . We perform further analysis only on the steady-state data. Fig. 3 shows snapshots of our steady-state data under three different conditions.

We compute the Fourier transform of our data using the fast Fourier transform (FFT) to obtain the amplitudes (u_0 , $u(x)$, and u_L , respectively) and phases (G_0 , $G(x)$, and G_L , respectively) of each of T_1 , T_2 and T_3 . The phase difference between T_1 (or T_3) and T_2 is used to calculate the

thermal diffusivity of the fluid by solving Eq. (11). It should be noted here that as a requirement for the FFT, we use only those numbers as our time periods which are powers of 2 [17].

Each measured point on the graphs shown in Figs. 4 and 5 represents the average of the values obtained from three experimental runs. Error bars are given with all measured results. The error bars for Figs. 4 and 5 are determined following the procedure described in [18], and are calculated based on three experimental runs for a 95% confidence level. Since the sample size, n , is less than 30 we use the Student- t distribution for calculation of precision errors, ε_P . According to this method $\varepsilon_P = t_{\alpha, \nu} \cdot (S_x / \sqrt{n})$ where $t_{\alpha, \nu}$ is the value of the Student- t distribution for a confidence level of α and a degree of freedom of ν , S_x the

standard deviation of the sample and n is the sample size. Here, $\nu = n - 1$ and n is the number of experimental runs.

6. Results and discussion

6.1. Evaluation of the experimental technique and the parameters involved

The most important parameters involved in this technique are the time period (t_p) and the amplitude of the temperature oscillation. If t_p is small, the oscillation would die out in the middle and no useful measurement can be done. On the contrary, if t_p is too large, there would be no phase difference between T_1 (or T_3) and T_2 . Additionally, if the amplitude is too high, onset of natural convection might take place and consequently the result will be erroneous. On the other hand, the amplitude should be large enough so that the sinusoidal pattern is discernible. The time period should be larger than the system response time, $[\pi(L/2)^2]/\alpha$ so as to allow the temperature wave to propagate up to the midpoint of the cylindrical volume along the length. However, this is a necessary condition for obtaining a measurable temperature wave at the midpoint of the volume, but not a sufficient one. As can be observed from Eq. (12), $u_{L/2}$ depends on both the amplitude and the time period of the oscillation used. We performed systematic experiments to obtain a quantitative measure of these effects and the results are summarized in Table 1.

In Fig. 3 we show examples of three different situations: Fig. 3(a) shows a no-oscillation situation at thermocouple T_2 , Fig. 3(b) presents a nice waveform with clear phase difference, and Fig. 3(c) shows a situation where there is no discernible phase difference.

Subsequently, we analyze the data and calculate the thermal conductivity values for water at 20 °C using a density of 998.4 kg m⁻³ and a heat capacity of 4.183 kJ kg⁻¹ K⁻¹. As can be seen in Fig. 4, there is no clear winner among the time periods of 128 s, 256 s and 512 s, and also among the amplitudes of 0.5 °C, 1.0 °C, and 1.5 °C. Since, as per the summary of Table 1, time periods of 128 s and 512 s are borderline cases, we choose to use 256 s as the time period for our subsequent measurements. On the amplitude side, 1.5 °C is the highest we can use to avoid natural convection. To be on the safe side, we choose to use 1.0 °C.

6.2. Validation of our experimental setup: thermal conductivity of deionized water

The next step was to validate our experimental setup by measuring the temperature dependence of the thermal conductivity of pure (deionized) water and comparing our results with tabulated data [16]. The result is shown in Fig. 5. The measured values are within 2% of the tabulated values, and therefore we conclude that one can obtain fairly accurate results using our experimental setup and the temperature oscillation technique.

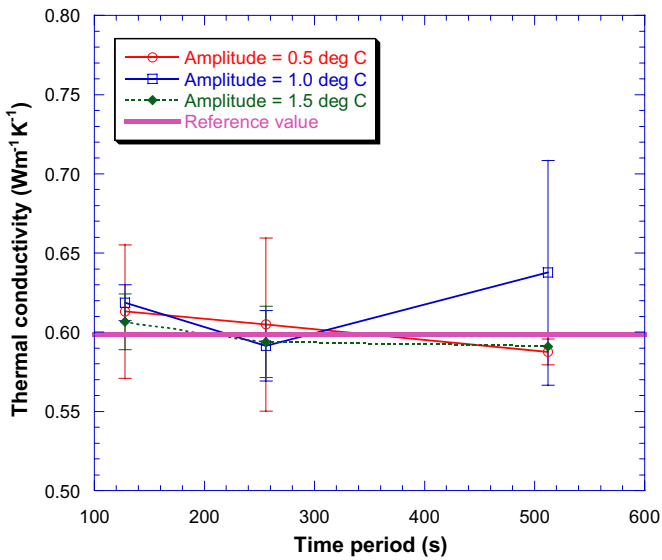


Fig. 4. Thermal conductivity of deionized water, compared to tabulated values [16], at 20 °C.

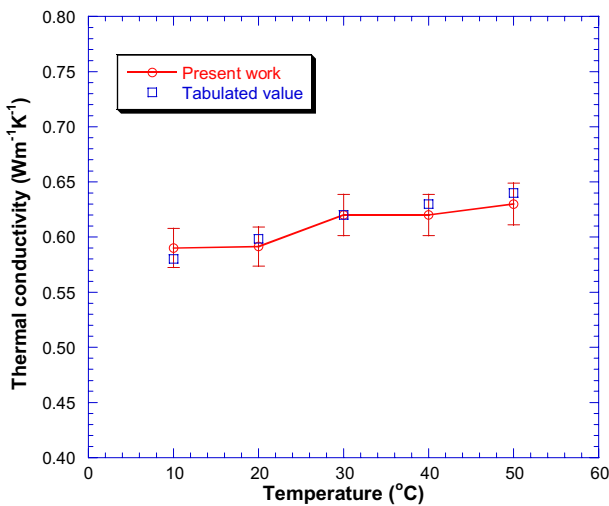


Fig. 5. Comparison between the measured thermal conductivity of deionized water and tabulated values [16] at different temperatures.

Table 1
Summary of the effect of amplitude and time period on the room-temperature measurement of the thermal diffusivity of deionized water

Amplitude (°C) Time period (s)	0.1	0.5	1.0	1.5
16	No oscillation captured by the middle thermocouple			
32	Nearly no oscillation captured by the middle thermocouple			
64	Slight oscillation captured by the middle thermocouple – no result can be obtained			
128	Irregular oscillation on all the thermocouples	Nice waveform on all the thermocouples with visible phase difference between the middle and the end thermocouples - the results need to be studied to compare these combinations		
256				
512				
1024	No visible phase difference	Not performed	Not performed	

7. Conclusions

Measurement of the thermal conductivity of pure water and fluids having $[\beta/(\alpha\nu)]$ values similar to that of water using the temperature oscillation technique is difficult if one does not choose the parameters judiciously, but it is a simple and reliable technique once appropriate values of the concerned parameters are determined. A diameter-to-length ratio of 8 for the cylindrical chamber, and using 6-mm-thick aluminum reference plates at each end of the cylinder, can make the heat conduction inside the fluid one-dimensional. For fluids having $[\beta/(\alpha\nu)]$ values similar to that of water, a cylindrical chamber of length less than 8.7 mm, a time period falling between 128 s and 512 s, and an imposed sinusoidal temperature amplitude in the range of 0.5 °C to 1.5 °C are recommended. For fluids having $[\beta/(\alpha\nu)]$ values largely different from that of water, an analysis similar to what has been performed in the present work can be done.

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